

Merging History Trees for Dark Matter Halos: Tests of the Merging Cell Model in a CDM Cosmology

B. Lanzoni¹, G. A. Mamon^{1,2}, B. Guiderdoni¹

¹ *Institut d'Astrophysique de Paris, (CNRS UPR 0341), Paris, France*

² *DAEC (CNRS UMR 8631), Observatoire de Paris, Meudon, France*

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ABSTRACT

The merging history of dark matter halos is computed with the Merging Cell Model proposed by Rodrigues & Thomas (1996). While originally discussed in the case of scale-free power spectra, it is developed and tested here in the framework of the cold dark matter cosmology. The halo mass function, the mass distribution of progenitors and child halos, as well as the probability distribution of formation times, have been computed and compared to the available analytic predictions. The halo auto-correlation function has also been obtained (a first for a semi-analytic merging tree), and tested against analytic formulae. An overall good agreement is found between results of the model, and the predictions derived from the Press & Schechter theory and its extensions. More severe discrepancies appear when formulae that better describe N-body simulations are used for comparison. In many instances, the model can be a useful tool for following the hierarchical growth of structures. In particular, it is suitable for addressing the issue of the formation and evolution of galaxy clusters, as well as the population of Lyman-break galaxies at high redshift, and their clustering properties.

Key words: cosmology: theory, dark matter – galaxies: halos, formation – galaxies: clusters: general

1 INTRODUCTION

In the basic picture of the formation of cosmic structures, the Universe is dominated by a dark matter (DM) component, and small perturbations in the initial density field grow in amplitude proportionally to a linear growth factor, until they approach unity. Then, non-linear effects dominate their evolution, and the regions stop expanding with the Universe, collapse, and virialise, thus forming DM halos. In hierarchical scenarios, like the cold dark matter (CDM) model, small-scale inhomogeneities collapse first, and then aggregate via merging to generate larger structures. Since galaxies form by the collapse and cooling of baryonic gas within DM halos, and their history is greatly influenced by that of their surrounding halos (e.g., Lemson & Kauffmann 1999), it is important to understand how these objects form and evolve with time.

The most realistic way for following the history of DM halos is by means of N-body simulations, but they require huge amounts of RAM memory and are computationally expensive. Therefore, they are often limited to a modest dynamic range, and to few different cosmological scenarios.

The simplest alternative approach is to consider only the linear regime of growth of density fluctuations, and de-

scribe the non-linear evolution and collapse by means of the spherical ‘Top-Hat’ model (Gunn & Gott 1972). In this formalism, the formation of a DM halo of mass M at redshift z is described by identifying in the initial density field smoothed on a scale M , and *linearly* extrapolated to redshift z , a region having overdensity equal to a given threshold value. Starting with Gaussian initial conditions, Press & Schechter (1974, hereafter PS) interpreted the probability of finding such regions as the number density of halos of mass M , that formed at redshift z (see Sec. 3.1). The PS mass function has been extensively tested against N-body simulations, and has been found to be in reasonably good agreement with numerical results (e.g., Efstathiou et al. 1988; Carlberg & Couchman 1989; Lacey & Cole 1994; Gelb & Bertschinger 1994). However, systematic deviations both at small and high masses have been recognised, with the PS formula predicting too many low mass halos, and underestimating the number of massive objects (e.g., Jain & Bertschinger 1994; Gross et al. 1998; Somerville et al. 1998; Tormen 1998; Lee & Shandarin 1999; Sheth & Tormen 1999; and references therein). A better agreement with numerical results is obtained when ellipsoidal rather than spherical collapse models are considered (Monaco 1995, 1997a, b; Bond & Myers 1996; Audit, Teyssier & Alimi 1997, 1998; Lee &

Shandarin 1998, 1999, hereafter LS98 and LS99; Sheth & Tormen 1999; Sheth, Mo, & Tormen 1999; and references therein).

Extensions of the PS theory (Bower 1991; Bond et al. 1991) follow the redshift evolution of the halo population as a whole, by deriving the conditional probability of finding progenitors of mass M_p at redshift z_p , given their child halos of mass M_o at z_o , and vice versa (see Sec. 3.2). By means of the extended Press & Schechter (EPS, hereafter) theory, the distribution of halo formation and survival times, as well as their merger rate, can also be obtained (Lacey & Cole 1993, 1994, hereafter LC93 and LC94; see also Sec. 3.3). The few comparisons between these analytic predictions and numerical results reveal a general good agreement, even if discrepancies similar to those of the mass function have been pointed out (LC94; Somerville et al. 1998; Tormen 1998).

Still based on the EPS theory, analytic predictions for halo bias in the Lagrangian space of initial conditions have been obtained (Mo & White 1996, MW96 hereafter; Catelan et al. 1998, CLMP; Porciani et al. 1998; Sheth & Lemson 1999b; Sheth, & Tormen 1999; Sheth, Mo, & Tormen 1999; and references therein). The halo auto-correlation function $\xi_{hh}(r)$ is then the product of the halo bias with the correlation function of the underlying matter. The predicted ξ_{hh} is in good agreement with that in N-body simulations for massive objects, but its amplitude is too large for low-mass halos (Porciani, Catelan & Lacey 1999, PCL hereafter; Jing 1999; Sheth & Tormen 1999). However, Jing (1999) propose an empirical fitting formula (see Sec. 3.5) that provides a good description of halo clustering on the whole range of masses (see also Sheth & Tormen 1999; Sheth, Mo, & Tormen 1999).

While the PS and EPS formalisms describe the mean statistical properties of the population as a whole, several models of the individual merger history of DM halos have been proposed (Cole & Kaiser 1988; Kauffmann & White 1993, KW93 hereafter; Rodrigues & Thomas 1996, RT96 hereafter; Somerville & Kolatt 1999; Sheth & Lemson 1999a). Each model presents some advantages and some drawbacks with respect to the others. For example, the “block model” of Cole & Kaiser (1988) partly retains the spatial information, but it is affected by the discretisation of both halo masses (in powers of two), and positions. The KW93 merging tree presents a more continuous spectrum of masses, but a grid of collapse redshifts is imposed, and the relative positions of halos are unknown. Moreover, it reproduces exactly the mean progenitor mass distribution, but mass conservation is enforced only approximately, while the opposite holds in the model of Somerville & Kolatt (1999).

In this paper, we focus on the “Merging Cell Model” (MCM) ^{*} proposed by Rodrigues & Thomas (1996), which has the same characteristics of simplicity and speed as the other merging tree algorithms, but also presents some major advantages. Since it is based on an actual realisation of the initial density field, it is much closer to the spirit of N-body simulations, thus allowing direct comparisons with numerical results, and it also seems to take into account the spatial correlations of density fluctuations (Nagashima & Gouda 1997). Moreover, no specific collapse times are im-

posed *a priori*, and halos form with a continuous spectrum of masses, and a variety of (Lagrangian) shapes. Also the spatial information about the relative location of halos is retained by construction, thus allowing to study their clustering properties. While in the original paper, the authors only discuss the halo mass function in the case of scale-free power spectrum, here we consider the more realistic standard CDM (SCDM) cosmology (see Sec. 3). Moreover, we test the model reliability also in terms of the mass distribution of progenitor and child halos, the behaviour of the largest progenitor mass as a function of redshift, and the probability distribution of formation times. For the first time for a semi-analytic merging tree, also the halo two-point correlation function is computed, and we test it against theoretical predictions. We outline the method in Sec. 2, define these quantities and compare them to the analytic predictions in Sec. 3. Discussion and conclusions are presented in Secs. 4 and 5, respectively.

2 THE ALGORITHM

2.1 Basic Principles

At an ‘initial time’ t_i , consider the density field $\rho(\mathbf{x}, t_i)$ of the Universe characterised by a mean value $\bar{\rho}(t_i)$, and small perturbations $\delta(\mathbf{x}, t_i) = \rho(\mathbf{x}, t_i)/\bar{\rho}(t_i) - 1$. Through gravitational instability, the amplitude of density fluctuations start growing proportionally to a linear growth factor $D(t)$, i.e., $\delta(\mathbf{x}, t) = \delta(\mathbf{x}, t_i) \times D(t)/D(t_i)$. Such a linear growth law strictly holds only when perturbations are much smaller than unity, but it is useful to extrapolate it also into the non-linear regime. In fact, the ‘Top-Hat’ model (Gunn & Gott 1972) shows that the formation of a bound virialised object of mass M occurs at the time t_f when the density contrast of a spherical region in the initial density field, smoothed at a scale M , reaches a critical value δ_c . This in turn corresponds to a value $\delta_c^{\text{lin}}(t_f)$ of the density field *linearly* extrapolated to that time, or to a value $\delta_c^{\text{lin}}(t_0)$ if the extrapolation is carried on until the present epoch t_0 . For an Einstein-de Sitter Universe, $D(t) \propto (1+z)^{-1}$, $\delta_c \simeq 178$, $\delta_c^{\text{lin}}(z_f) \simeq 1.686$, and $\delta_c^{\text{lin}}(z=0) \simeq 1.686(1+z_f)$. It is therefore sufficient to know the values of the density field linearly extrapolated to $z=0$, for determining the formation epochs of DM halos.

Because this is the approach we adopt in the paper, we choose henceforth to change the notation and we denote the density field linearly extrapolated to $z=0$ as δ . Therefore, $\delta_c = 1.686$, and the collapse redshift of halos is $z_f = \delta/\delta_c - 1$.

2.2 The Method

The MCM is based on an actual realisation of the density field, obtained with a standard initial condition generator, by Fourier transforming waves of random phase and amplitude drawn from a Gaussian distribution of zero mean, and variance given by the chosen power spectrum.

A value of the density contrast δ is assigned to each of the L^3 *base cells* (*bcs*) composing a periodic cubic box of side L (for simplicity, $L=2^l$, where l is a positive integer). Density fluctuations are then averaged within cubic *blocks* of side 2, 4, 8, ..., L . At each of these smoothing levels,

^{*} as first coined by Nagashima & Gouda (1997).

a set of 8 overlapping grids displaced one relative to another by half a block length in each coordinate direction is used. This ensures that the density peaks are always approximately centred within one of the blocks in each smoothing hierarchy. At this point, one has a total of $(15L^3 - 8)/7$ base cells and cubic blocks, with side ranging from 1 to L , mutually overlapping in the volume of the box. Each of them is characterised by a value of the density contrast δ .

All base cells and blocks are then ordered in a single list in terms of decreasing δ (or, correspondingly, decreasing collapse redshift). The largest value of δ in the list fixes the earliest z_f of the realisation. It usually corresponds to a base cell, which thus becomes the first collapsed object (i.e., the first *halo*). All the elements of the list are then analysed one after the other from early times to the present, and the specific base cell or cubic block under investigation is called *investigating region*. Whether the investigating region can collapse and give rise to a new halo or not is decided by the following rules:

- (i) an investigating region that does not overlap with any other pre-existing halo collapses and forms a new halo;
- (ii) if there exist two halos, each of them containing half of the investigating region, the latter cannot collapse. This is to avoid the formation of very elongated structures in linking together adjacent halos without the collapse of any new matter. If instead there exists only one halo containing half (or more) of the investigating region, the latter collapses and merges with it, thus forming a new halo;
- (iii) after taking into account condition (ii), if the investigating region overlaps with at least half of one (or more) pre-existing halo(s), it collapses and merges with it (them), thus forming a new halo.

Note that in the Lagrangian space of initial conditions, mass and volume are equivalent quantities ($M = \bar{\rho} V$), and ‘merging’ together the investigating region with one or more pre-existing halos does not mean summing up their masses. Instead, the mass (volume) of the new resulting halo is that of the old ones plus the fraction of the investigating region that does not overlap with any already pre-existing object.

Thanks to the use of overlapping grids and merging criteria, halos of a large variety of shapes and masses are obtained. The model also contains information on the relative locations of halos, since their positions within the box are known, and the effects of discretisation are expected to be smaller than in the block model. Moreover, because halo formation times are given by the density contrasts in the list, they span a continuous range of values. As a drawback, the overdensity of the investigating region (which fixes the collapse redshift of the new forming object) is not necessarily equal to the mean overdensity ($\bar{\delta}_h$) of the resulting halo. Therefore, the position of an object in the merging tree occasionally differs from that predicted by the linear theory, i.e., the assigned z_f is not exactly equal to $\bar{\delta}_h/\delta_c - 1$, as it should (see Sec. 3.2 in RT96). Finally, the ‘linking’ and the ‘overlapping’ conditions [criteria (ii) and (iii), respectively] are reasonable but arbitrary, and different choices would result in different mass functions, as discussed by Nagashima & Gouda (1997). Yet, it is not clear which are the ‘best’ criteria. Thus, we will adopt the original conditions (ii) and (iii) throughout the paper.

3 TESTS OF THE ALGORITHM

The MCM is based on the linear theory of growth of density fluctuations, and it uses simplified criteria to describe the formation and merging history of DM halos. It is therefore necessary to test its reliability by comparing its results against those of N-body simulations that directly take into account the gravitational interactions between DM particles, and are much more realistic in following the dynamics in the non-linear regime.

The model is required to correctly describe not only the population of halos at a given redshift, but also how this population evolves with time.

As a first test, the cumulative and the differential mass functions in the case of a scale-free power spectrum with spectral index $n = 0$, and -2 have been computed and compared to those in the original paper (Figs. 4 and 5 in RT96). A remarkable agreement has been found.

Here we consider the SCDM cosmology, and perform several tests against the available analytic formulae, to verify the reliability of the model results. We set the Hubble constant to $H_0 = 100 h \text{ km s}^{-1} \text{ Mpc}^{-1}$, $h = 0.5$. The total and baryonic density parameters are $\Omega_0 = 1$ and $\Omega_b = 0.05$ respectively, while that corresponding to the cosmological constant is $\Omega_\Lambda = 0$. The transfer function of Bardeen et al. (1986) is adopted, and the power spectrum is normalised so that the mass variance on scale $8 h^{-1} \text{ Mpc}$ is equal to $\sigma_8 = 0.67$.

For a 256^3 base cell realisation in a cubic box of $L=100 \text{ Mpc}$ side (i.e., $50 h^{-1} \text{ Mpc}$, and a total mass of about $3.5 \times 10^{16} h^{-1} \text{ M}_\odot$), the base cell mass is of about $2 \times 10^9 h^{-1} \text{ M}_\odot$, and the resulting most massive halo typically has a mass of about $5 \times 10^{14} h^{-1} \text{ M}_\odot$ (but see Sec. 3.1). The CPU time on a 500 MHz DEC Alpha workstation is only roughly 25 min, and typically 700 MB of RAM memory are required.

In the following sections, results averaged over 10 different realisations are presented, and error bars correspond to the standard deviations of the 10 run sample.

3.1 Mass Function

The differential mass function of halos, is defined as the comoving number density of halos with mass in the range $[M, M+dM]$ at redshift z . This is shown for logarithmic mass interval in the histograms of Fig. 1 (left-hand panels), for $z = 0$ and $z = 3$.

Also shown for comparison as dotted lines are the corresponding predictions of the PS theory (e.g., LC94):

$$\frac{dn}{d \ln M}(M, z) = \sqrt{\frac{2}{\pi}} \bar{\rho} \frac{\delta_c(z)}{\sigma^2(M)} \left| \frac{d\sigma}{dM} \right| \exp \left[-\frac{\delta_c^2(z)}{2\sigma^2(M)} \right], \quad (1)$$

where $\delta_c(z) = \delta_c \times (1+z)$, and $\sigma(M)$ is the mass variance of the linearly extrapolated (to $z = 0$) density field smoothed on scale M . In this paper, $\sigma(M)$ is always computed by a fitting formula analogous to that proposed by White & Frenk (1991), with errors smaller than 8% on mass scales ranging from 10^9 M_\odot to 10^{15} M_\odot .

Right panels show the cumulative mass fraction for the same redshifts, i.e. the fraction of the total mass which is in halos of mass above M , at redshift z .

At $z = 0$ the overall agreement is good over a large range of masses, but a lack of objects at the two ends of the

mass function is evident. At small masses, the problem seems to be inherent to the method, since this is also the case for scale-free power spectra (RT96; Nagashima & Gouda 1997). This is a drawback of the model, which limits the reliable dynamical range, and it is probably due to the adopted criteria for the formation and merging of halos. In fact, as detailed in Sec. 2, the “bricks” for the construction of halos are base cells and blocks composed by 8^i *bcs* ($i = 1, 2, \dots$). Thus, an object of less than 8 *bcs* can only result if an *investigating region* partly overlaps with a pre-existing halo, but does not merge with it, and the fraction of its non-overlapping volume (which gives rise to the new halo) is less than 8 *bcs*. In practice, the model requires that some “particular” conditions happen in order to form halos of mass between 2 and 7 *bcs*, thus explaining the underproduction of this kind of objects in the resulting mass function. The lack of high-mass structures instead, is partly inherent to the method, partly due to a statistical fluctuation (in different realisations in fact the problem is more or less severe). At high redshift the model always tends to produce a larger number of intermediate mass halos, and less massive objects than predicted by the PS theory.

These discrepancies appear to be even more severe if compared to results of N-body simulations. It has been recently shown, that the PS mass function already tends to predict fewer high-mass halos, and more low mass objects than those found in the simulations (e.g., Jain & Bertschinger 1994; Gross et al. 1998.; Somerville et al. 1998; Tormen 1998; LS99; Sheth & Tormen 1999). An analytic formula which better agrees with numerical results has been obtained by LS98, based on a nonspherical model for the collapse of a perturbation, in the frame of the Zel’dovich approximation (but see also Monaco 1995, 1997a,b; Audit, Teyssier & Alimi 1997; Bond & Myers 1996; Sheth & Tormen 1999; Sheth, Mo, & Tormen 1999).

In this formalism, the displacement of a particle due to the surrounding density field, is simply computed from the perturbation potential Ψ generated by the distribution of particles in the *initial* conditions. The mass density can therefore be expressed as a function of the three eigenvalues of the deformation tensor (defined as the second derivative of Ψ), and a virialised bound object forms when the smaller one (λ_3) is positive. The idea is therefore to substitute the collapse condition of the spherical Top-Hat model ($\delta = \delta_c$), with an analogous one for λ_3 : a DM halo of mass M forms when this eigenvalue reaches a critical value λ_{3c} in a region of the linearly extrapolated density field, smoothed on a scale M . The resulting mass function is:

$$\frac{dn_{LS}}{d \ln M}(M, z) = \frac{25\sqrt{10}}{2\sqrt{\pi}} \frac{\bar{\rho}}{M} \left| \frac{d \ln \sigma}{d \ln M} \right| \frac{\lambda_{3c}(z)}{\sigma} \times f[x], \quad (2)$$

with $x = \lambda_{3c}(z)/\sigma$, and:

$$f[x] = \left[\frac{5}{3}x^2 - \frac{1}{12} \right] \exp \left[-\frac{5}{2}x^2 \right] \operatorname{erfc} \left[\sqrt{2}x \right] + \\ + \frac{\sqrt{6}}{8} \exp \left[-\frac{15}{4}x^2 \right] \operatorname{erfc} \left[\frac{\sqrt{3}}{2}x \right] + \\ - \frac{5\sqrt{2\pi}}{6\pi} x \exp \left[-\frac{9}{2}x^2 \right],$$

where $\sigma = \sigma(M)$, $\operatorname{erfc}(x)$ is the complementary error func-

Figure 1. *Left panels:* comoving number density in units of Mpc^{-3} , per logarithmic mass interval, of halos of mass M at redshifts $z = 0$ (*upper panel*) and $z = 3$ (*lower panel*), as a function of M/M_\odot . Masses are also shown in units of base cells on the top of the figure. Results averaged over 10 realisations of the MCM are displayed in the histograms. Error bars show the standard deviations of the 10 run sample. The Press & Schechter (1974) predictions [eq.(1)] and the Lee & Shandarin (1998) mass function [eq.(2)] are plotted as *dotted* and *solid* curves, respectively. *Right panels:* cumulative mass fraction of halos with mass larger than M at redshifts $z = 0$ and $z = 3$, as a function of M . Results from the MCM (*thick solid line*) are compared to the PS and the LS98 predictions (*dotted* and *solid* curves, respectively).

tion, and the critical value for λ_3 has been empirically chosen to be $\lambda_{3c}(z) = 0.37(1+z)$.

Figure 1 shows that in comparison to the LS98 mass function (solid curves), the MCM presents an excess of small objects, and a significant underproduction of high-mass halos, especially at high redshift.

3.2 Conditional Mass Functions

In this section we analyse how the population of halos identified at a given time has changed with respect to a different epoch.

Figure 2 shows the mass fraction of halos of mass M_o at redshift z_o , that has already settled at redshift z_p in progenitors with masses between M_p and $M_p + dM_p$. Child halos have been selected at $z_o = 0$ and have masses M_o in the range $M_o/M_\odot = [10^i, 10^{i+1}]$, where, from the top to the bottom panel, $i = 11, 12, 13, 14$. The mass distribution of their progenitors at $z_p = 1$ is shown in the left panels, that for $z_p = 3$ is plotted on the right-hand panels.

The analytic prediction for the distribution of progenitor masses is given by (e.g., Bower 1991, Bond et al. 1991):

$$\frac{df}{d \ln M_p}(M_p, z_p | M_o, z_o) = \frac{M_p}{\sqrt{2\pi}} \frac{\delta_{cp} - \delta_{co}}{(\sigma_p^2 - \sigma_o^2)^{3/2}} \left| \frac{d\sigma_p^2}{dM_p} \right| \times \exp \left[-\frac{(\delta_{cp} - \delta_{co})^2}{2(\sigma_p^2 - \sigma_o^2)} \right], \quad (3)$$

where $\delta_{cj} = \delta_c(z_j)$, $\sigma_j = \sigma(M_j)$.

Because numerical results shown in Fig. 2 have been obtained for ranges of M_o , the analytic formula has been computed in two different ways for an accurate comparison. In the MCM, the mass of a halo corresponds to the number of base cells that compose it. Thus, when expressed in these units, the mass M_o can only assume \mathcal{N} integer values in the range $[M_i, M_s]$, with $\mathcal{N} = (M_s - M_i + 1)$: i.e., $M_o = \{M_{ok} = M_i + k - 1, k = 1, \mathcal{N}\}$. Given the number $N(M_{ok})$ of child halos with mass equal to M_{ok} for each of its possible \mathcal{N} values, the mean weighted mass in the range is:

$$\overline{M}_o = \frac{\sum_{k=1}^{\mathcal{N}} N(M_{ok}) M_{ok}}{N_o}, \quad (4)$$

where N_o is the total number of halos in the chosen range of M_o . Dotted curves in Fig. 2 show results from eq.(3) computed for $M_o = \overline{M}_o$ (whose numerical values are listed in the figure caption). The abrupt fall down of dotted curves occurs at values of M_p near \overline{M}_o , because the progenitor mass obviously cannot be larger than that of its child halo. Since \overline{M}_o is lower than M_s in each panel, this explains why dotted curves are not as extended in M_p as the histograms are.

For a better comparison between numerical and analytic results, we have also computed the *average* progenitor mass distribution by summing up the \mathcal{N} single-mass distributions, each weighted with the fraction of mass in halos of mass M_{ok} [$N(M_{ok}) \times M_{ok}$]:

$$\left\langle \frac{df}{d \ln M_p} \right\rangle = \frac{\sum_{k=1}^{\mathcal{N}} f(M_{ok}) N(M_{ok}) M_{ok}}{\sum_{k=1}^{\mathcal{N}} N(M_{ok}) M_{ok}}, \quad (5)$$

where $f(M_{ok}) = (df/d \ln M_p)$ is the single-mass progenitor distribution for child halos of mass M_{ok} as given in eq.(3). M_p is obviously also a function of M_o , and it is required that $M_p \leq M_o$. The average progenitor mass distribution is plotted in Fig. 2 as solid curves. By construction, $\langle df/d \ln M_p \rangle$ is a sum of curves of the same kind of the dotted lines, with the sharp cut-off at progenitor masses very similar or equal to that of their child halos. This is the reason for the oscillations in the solid curves for values of M_p between M_i and M_s .

A lack of objects with masses between 2 and about 7 *bcs* is apparent, just as was the case for the mass function. The MCM also appears to systematically underproduce progenitors with mass similar to that of their child halos. At intermediate masses, an overall good agreement between the MCM results and the analytic predictions is found, with a possible slight overproduction of halos in the model. When compared to N-body simulations, these discrepancies may become more severe, since simulations appear to have fewer/more halos than predicted by EPS theory in the intermediate/high mass range (Somerville et al. 1998; Tormen 1998). For the less massive child halos (top panels), numerical and analytic results only agree over a small range

Figure 2. Progenitor mass distribution at redshifts $z_p = 1$ (left panels), and $z_p = 3$ (right panels), for child halos in four different mass ranges M_o at $z_o = 0$ (from top to bottom: $M_o/M_\odot = [10^i, 10^{i+1}[$, $i = 11, 12, 13, 14$). Average results from 10 realisations of the MCM are displayed in the histograms, and their standard deviation is shown as error bars. The average number of child halos found in each mass range is, from top to bottom: $N_o = 34839, 5545, 822$, and 67 . Dotted curves correspond to the progenitor distribution computed from eq.(3) for M_o equal to the mean mass in the corresponding range ($M_o = \overline{M}_o \simeq 2.7 \times 10^{11}$, 2.5×10^{12} , 2.3×10^{13} , and $1.5 \times 10^{14} M_\odot$). Solid curves are the average distribution in each mass range, computed from eq.(5).

of M_p . This is due to the lack of low mass objects, and to the fact that the minimum mass in the model is limited to 1 base cell, thus not allowing to accurately follow back in time the past history of small halos.

The reverse conditional probability that a halo of mass M_p at z_p is incorporated at a later time z_o in a halo of mass between M_o and $M_o + dM_o$, is shown per logarithmic mass interval in the histograms of Fig. 3. Progenitors of mass $M_p/M_\odot = [10^i, 10^{i+1}[$, with $i = 11, 12, 13$, are selected at $z_p = 1$ (left panels) and $z_p = 3$ (right panels), and the mass distribution is computed for their child halos at redshift zero. No results for progenitors with masses between 10^{14} and $10^{15} M_\odot$ are shown, because too few of them have already formed at redshift 1 and 3.

Given all the objects of mass M_p at z_p , the analytic prediction from the EPS theory for the mass distribution of their child halos at z_o , when expressed per mass logarithmic interval, is given by (e.g., LC94):

$$\frac{df}{d \ln M_o}(M_o, z_o | M_p, z_p) = \frac{M_o}{\sqrt{2\pi}} \frac{\delta_{co}(\delta_{cp} - \delta_{co})}{\delta_{cp}} \left| \frac{d\sigma_o^2}{dM_o} \right| \times \left[\frac{\sigma_p^2}{\sigma_o^2(\sigma_p^2 - \sigma_o^2)} \right]^{3/2} \exp \left[-\frac{(\delta_{co}\sigma_p^2 - \delta_{cp}\sigma_o^2)^2}{2\sigma_p^2\sigma_o^2(\sigma_p^2 - \sigma_o^2)} \right], \quad (6)$$

where the notation is the same as in eq.(3).

As before, eq.(6) has been computed in two different ways, in order to get an accurate comparison with the numerical distributions. Results for $M_p = \overline{M}_p$, the mean weighted progenitor mass in the range $[M_i, M_s]$ (analogous to \overline{M}_o), are plotted in Fig. 3 as dotted lines. A sharp cut off occurs for values of M_o near to \overline{M}_p , because child halos cannot be less massive than their progenitors. Since \overline{M}_p is larger than M_i in each panel, this explains why dashed lines are not as extended in M_o as histograms are. A more appropriate comparison between numerical and theoretical results is obtained if the *average* child mass distribution $\langle df/d \ln M_o \rangle$ is considered, instead of that relative to pro-

Figure 3. Mass distribution of child halos at redshifts $z_o = 0$, given the progenitors at $z_p = 1$ (*left panels*), and $z_p = 3$ (*right panels*) with masses, $M_p/M_\odot = [10^i, 10^{i+1}[$, $i = 11, 12, 13$, from *top to bottom*. Average results from 10 realisations of the MCM are plotted as histograms, and their standard deviation is shown as error bars. The number of progenitors found in each mass range at $z_p = 1$ is, from top to bottom: $N_p = 59898, 6696$, and 431 , while at $z_p = 3$, $N_p = 52213, 1724$, and 8 . *Dotted* curves refer to the children mass distribution computed from eq.(6) for the mean progenitor mass in the corresponding range (from top to bottom): $\bar{M}_p \simeq 2.6 \times 10^{11}, 2.2 \times 10^{12}$, and $1.7 \times 10^{13} M_\odot$ for progenitors at $z_p = 1$; $\bar{M}_p \simeq 2.3 \times 10^{11}, 1.9 \times 10^{12}$, and $1.5 \times 10^{13} M_\odot$ in the right panels. *Solid* curves are the average distribution in each mass range, computed in the same way as in eq.(5), as detailed in the text.

genitors with mean mass \bar{M}_p : $df(\bar{M}_p)/d\ln(M_o)$. The computation of $\langle df/d\ln M_o \rangle$ is analogous to that in eq.(5), and results are plotted in Fig. 3 as solid lines.

An overall agreement is found between MCM results and the EPS theory predictions, that in turn fit reasonably well N-body simulations (LC94). However an oscillating behaviour of the child halos mass distribution can be recognised in the histograms. Actually it is more evident when a different binning is used (here results are binned on a mass grid $M_o = 2^i \text{ bcs}$, $i = 0, 1, 2, \dots$), and it seems inherent to the method. Also the halo mass function and the progenitor distribution present analogous features, and oscillations appear to occur with peaks corresponding to the block masses of 8^i , $i = 1, 2, \dots \text{ bcs}$, and with troughs in between. Moreover, the same trend is found in the halo mass function for the scale free power spectrum (in particular for the spectral index $n = -2$, that fits the CDM spectrum over a significant range of masses; see RT96).

3.3 Largest progenitor history

By analysing the variation with redshift of the largest progenitor mass, information can be obtained on how halos build up in time, whether they preferably form via a continuous and slow accretion of small objects, or whether their mass suddenly increases because of nearly equal-mass merging events, or by mergers of several sub-units at the same time. A different behaviour is expected for halos of different masses, with larger objects preferably assembling at recent epochs, and smaller halos showing a more delayed and smooth evolution with time. This is shown for instance, in KW93, both from their merging tree model, and from N-body simulations (their Figs. 5 and 6, respectively).

We have looked at the past history of halos with cur-

Figure 4. History of the most massive progenitor of 30 halos selected at $z_o = 0$. The y -axis represents the ratio of progenitor mass M_p to final halo mass M_o . The four panels refer to four different values of M_o/M_\odot : $10^{11}, 10^{12}, 10^{13}, 10^{14}$.

rent mass $M_o = 10^{11}, 10^{12}, 10^{13}, 10^{14} M_\odot$, randomly selecting 30 objects for each value of M_o to show the scatter in the merging histories. The ratio between the mass of the largest progenitor M_1 , and that of its child halo M_o is plotted in Fig. 4, as a function of $1+z$. For all the masses, the expected trends are obtained, with larger halos preferably assembling through major mergers at low z , and smaller objects gradually forming in a smoother way by accreting small mass objects over a larger interval of time. A qualitative good agreement of both trends and scatters is also found between the present results and those of KW93. Moreover, halo collapse occurs at more recent epochs here, as expected when a SCDM cosmology is considered instead of an open model ($\Omega_0 = 0.2$ in KW93).

3.4 Formation redshift

In the hierarchical clustering scenario, massive halos form by accretion of lower mass structures. Therefore, their formation redshift is expected on average to be lower than that of small objects. Actually, because of the continuous evolution in mass due to the hierarchical nature of the process, the definition of ‘halo formation time’ is not straightforward. In this paper, we adopt the definition of LC93, as the time when half the mass of the halo is assembled, i.e., when a progenitor with mass equal to half or more that of its child halo appears for the first time.

Figure 5 shows the distribution of formation redshift for halos with mass $M_o = [10^i, 5 \times 10^i] M_\odot$, $i = 11, 12, 13, 14$ at $z = 0$. In agreement with results of the previous section, high mass objects tend to form at more recent epochs, while lower mass halos typically collapse earlier and over a larger interval of time. The mean formation redshifts for halos in the four mass ranges, from lower to higher M_o , are: $\bar{z}_f = 1.55, 1.03, 0.66, 0.46$.

As discussed in LC93 and LC94, the probability that a halo of mass M_o at redshift z_o has a progenitor with mass between $M_o/2$ and M_o at z_p , gives the probability that its formation epoch was earlier than z_p . In differential form, the probability distribution of formation redshifts is therefore given by:

$$\frac{dp}{dz_f}(z_f|M_o, z_o) = \int_{M_o/2}^{M_o} \frac{M_o}{M_p} \left[\frac{\partial}{\partial z_f} \left(\frac{df}{dM_p} \right) \right] dM_p, \quad (7)$$

Figure 5. Differential probability distribution of formation redshifts z_f for halos at $z_o = 0$ with masses $M_o/M_\odot = [10^i, 5 \times 10^i]$, $i = 11, 12, 13, 14$ (see labels), as a function of $(1 + z_f)$. Histograms and error bars result from the average over 10 realisations of the MCM. N_o labels the number of halos found at $z_o = 0$ in the corresponding mass range. *Solid curves* refer to the analytic prediction of the EPS theory computed from eq.(7), for $M_o = \bar{M}_o \simeq 2 \times 10^{11}, 2 \times 10^{12}, 1.7 \times 10^{13}, 1.4 \times 10^{14} M_\odot$.

where $df/dM_p = df(M_p, z_f | M_o, z_o)/dM_p$ is the progenitor mass distribution (see Sec. 3.2). Formation times computed by means of the previous formula are found to be in good agreement with N-body simulation results, except for halos on cluster scales that form earlier than predicted by the EPS theory (LC94; Tormen 1998; note however that these conclusions are drawn for scale-free power spectra only).

Once again, numerical results are derived for ranges of masses M_o , thus the analytic prediction for z_f has been computed in the same way as discussed in Sec. 3.2. In this case however, the probability distribution for the average mass $dp(z_f | \bar{M}_o, z_o)/dz_f$, and the *average* probability distribution $\langle dp/dz_f \rangle$ are almost indistinguishable in the chosen range of M_o . Only the former is therefore shown in Fig. 5, where solid curves correspond to eq.(7) solved for $M_o = \bar{M}_o \simeq 2.2 \times 10^{11}, 2 \times 10^{12}, 1.7 \times 10^{13}, 1.4 \times 10^{14} M_\odot$. Within the error bars, a very good agreement is found for the most massive objects. For intermediate mass halos, the epoch when they first appear, as well as the rising of the probability distribution with decreasing z is well reproduced by the MCM. However, they do not present the expected peak of formation epoch, but instead still form at very recent times, in contradiction with the expectations of the EPS theory. A severe disagreement is found for small objects, with the departure of MCM relative to the EPS theory going in the opposite sense. Low mass halos in fact preferentially collapse and stop forming at earlier epochs than predicted, with a peak of formation at about $z_f = 1.6$, instead of $z_f = 0.85$. No significant improvements are obtained if different values for the collapse threshold δ_c are adopted. This confirms once more that the history of low mass objects is not well followed in the model.

3.5 Two-point Correlation Function

Since the relative positions of halos within the box are known by construction, the MCM also contains information about their spatial distribution. We have computed the two-point autocorrelation function of DM halos, by counting the num-

ber of objects separated by a distance r , and comparing it to the value expected for a Poissonian distribution:

$$\xi(r) = \frac{N_{DD}(r)}{N_{RR}(r)} - 1, \quad (8)$$

where $N_{DD}(r)$ is the number of pairs whose geometric centres are separated by a distance between r and $r + dr$, and $N_{RR}(r)$ is the same quantity if halos were randomly distributed in the same volume: $N_{RR}(r) = (1/2)N_o^2(dV/V)$, where N_o is the total number of halos, dV is the volume of the shell at r with thickness dr , and V is the total volume of the box.

Results for halos selected in four mass ranges at $z = 0$, 1 and 3 are shown as circles in Figs. 6, Fig. 7, and Fig. 8, respectively. Note that points are found at separations smaller than the typical halo sizes (marked by the vertical thick lines in plots). This is a consequence of the nonspherical shape of halos in the MCM, allowing the distance between two centres to be smaller than the spherical radius R artificially attributed to each object in this computation.

Using an approach based on the EPS theory, CLMP give an analytic formula for the halo two-point correlation function, which is valid for separations r larger than R . In particular, when $r \gg R$, the correlation function of objects of mass M identified at redshift z , can be expressed as:

$$\xi_{hh}(r, M, z) = b_1^2(M, z) \xi_m(r, z) + \frac{1}{2} b_2^2(M, z) \xi_m^2(r, z) + \dots, \quad (9)$$

where $\xi_m(r, z)$ is the matter correlation function (the Fourier transform of the power spectrum) linearly extrapolated to redshift z . The linear bias function $b_1(M, z)$ was already obtained by MW96 using a different approach still based on the EPS theory. It is given by:

$$b_1(M, z) = \frac{\delta_c}{\sigma^2(M, z)} - \frac{1}{\delta_c}, \quad (10)$$

where $\sigma(M, z)$ is the mass variance linearly extrapolated to redshift z : $\sigma(M, z) = \sigma(M) (1 + z)^{-1}$. CLMP show that the second order bias factor is:

$$b_2(M, z) = \frac{1}{\sigma^2(M, z)} \left[\frac{\delta_c^2}{\sigma^2(M, z)} - 3 \right]. \quad (11)$$

If the typical nonlinear mass $M_*(z)$ for dark matter halos is defined as $\sigma[M_*(z), z] = \delta_c$, it results from eq.(10) that the first order bias vanishes for $M=M_*$, and ξ_{hh} is then determined by the second order term only. For redshifts $z = 0, 1, 3$, the values of M_* are $3.4 \times 10^{13}, 1.2 \times 10^{12}$, and $6.2 \times 10^9 M_\odot$, respectively.

When a finite range of halo masses is considered instead of a single value of M , the theoretical halo correlation functions can still be estimated by eq.(9), with the two bias factors replaced by their mean values in the mass interval, weighted by the mass function $n(M, z) = dn/dM$:

$$\bar{b}_i = \frac{\int_{M_i}^{M_s} b_i(M, z) n(M, z) dM}{\int_{M_i}^{M_s} n(M, z) dM} \quad i = 1, 2. \quad (12)$$

Long dashed curves in Figs. 6, Fig. 7, and Fig. 8 have been computed by means of eq.(9)–(12) for the corresponding mass ranges and redshifts. Also shown for comparison are

the linear mass correlation function at each redshift (dotted lines), and ξ_{hh} computed with the linear term in eq.(9) only (dashed–dotted lines). For all redshifts and halo masses, the autocorrelation function derived from the MCM is in a remarkable good agreement with the predictions of the EPS theory. Even if the analytic formula for ξ_{hh} has been obtained in the limit of separations much larger than R , a reasonable agreement is also found when this condition is not exactly satisfied. Moreover, results of the MCM are well described by the linear bias relation also in the (slightly) non-linear clustering regime (i.e., for separations where $\xi_m(r)$ is slightly larger than unity), and thanks to the second order term, eq.(9) still provides a very good description of the model correlation function, even for masses near M_* , where b_1 vanishes.

Such an agreement between the MCM results and analytic predictions derived from the EPS theory only ensures the reliability of the model in correctly taking into account the clustering of high mass ($M \gtrsim M_*$) halos, but it also highlights its limitations for small objects. Indeed, accurate comparisons with N-body simulations show that the correlation function given by eqs.(9)–(12) correctly describes numerical results for halos with masses larger than M_* , but significantly overestimates the clustering of small mass objects (PCL; Jing 1999; Sheth & Tormen 1999; Sheth, Mo, & Tormen 1999). For $M < M_*$, the analytic bias factor b_1 is significantly lower (more negative) than that found in numerical simulations, whereas a better fit to the N-body Lagrangian correlation function (with errors within the 15% for a CDM cosmology) is obtained by means of the linear term in eq.(9), with b_1 replaced by (Jing 1999; but see also Sheth, Mo, & Tormen 1999):

$$b_J(M, z) = \left[\frac{\sigma^4(M, z)}{2\delta_c^4} + 1 \right]^{(0.06-0.02n)} \times [1 + b_1(M, z)] - 1, \quad (13)$$

where n is the index of the power spectrum $P(k)$, computed as:

$$n = \frac{d \ln P(k)}{d \ln k} \Big|_{k=2\pi/R}. \quad (14)$$

The correlation functions computed as $\xi_{hh} = \bar{b}_J^2 \xi_m$ are plotted in Figs. 6–8 as solid curves. Relative to these N-body based correlation functions, the MCM overestimates the clustering of halos on the low mass ($M < M_*$) regime.

4 DISCUSSION

As far as the halo mass function, and the conditional probability distribution of progenitor and child halos are concerned, a good general agreement between MCM results and PS and EPS analytic formulae is found, but an underproduction of low-mass objects in the model is apparent. This limits the mass resolution of the MCM to a minimum of 8 base cells ($3 \times 10^{10} M_\odot$, for the present choice of cosmological parameters and box size). Compared to the mass function from N-body simulations (well described when a nonspherical model for the collapse of density fluctuations is considered; see LS98; LS99; Sheth, Mo, & Tormen 1999, and references therein), the MCM produces a significantly lower number of high-mass halos, especially at early times. Since a finite box is used for representing the Universe, the

Figure 6. Auto-correlation function of halos with mass M selected at redshift $z = 0$. Each panel refers to a different mass range: $M/M_\odot = [10^i, 5 \times 10^i]$, with $i = 11, 12, 13, 14$ (see labels). Average results and standard deviation from 10 MCM realisations are plotted as *circles* and error bars, and the average number of halos found for each mass range is also indicated as N . Separations are in units of the box length ($L=100$ Mpc = 256 *bcs*). The three *vertical thick lines* mark the typical Lagrangian radius R of halos in the given range of M : the two shorter ones correspond to the minimum and the maximum mass in the range, while the longer one refers to the mean mass in the interval, weighted by the mass function. *Long dashed* curves have been computed by use of the linear and the second order bias factors [Catelan et al. 1998; see eqs.(9)–(12)], and the values of \bar{b}_1 and \bar{b}_2 are labelled in each panel. Also shown are the linear mass correlation function at the given redshift (*dotted* curves), and ξ_{hh} computed with the linear bias only, as first discussed by Mo & White (1996; *dashed–dotted* lines). *Solid* curves corresponds to the correlation functions computed as $\xi_{hh} = \bar{b}_J^2 \xi_m$, where the value of \bar{b}_J (see label) is derived from Jing’s formula [eq.(13)], which provides a good fit to N-body simulations. The first order bias vanishes at $M_* \simeq 3.4 \times 10^{13} M_\odot$.

Figure 7. The same as in Fig. 6, but for halos selected at $z = 1$. No results for halos with mass in the range $[10^{14}, 5 \times 10^{14}] M_\odot$ are shown because only 5 of them have already formed at this epoch. At $z = 1$, $M_* \simeq 1.2 \times 10^{12} M_\odot$.

effective amplitude of the mass variance on large scales is smaller than that expected from the input power spectrum, used in the computation of the analytic formulae. Such an effect may in part be responsible for the underproduction of high-mass halos in the MCM with respect to theoretical predictions. Also changing the *linking* and the *overlapping* conditions (see Sec. 2.2) helps obtaining larger mass halos, especially at high z , but it is not clear which are the best criteria (see also Nagashima & Gouda 1997). Moreover, oscillations in the mass functions occurring at block masses

Figure 8. The same as in Fig. 6, but for halos selected at $z = 3$. No results for larger masses are shown because too few, or no high-mass halos, have already formed at this redshift. Here, $M_* \simeq 6.2 \times 10^9 M_\odot$.

of 8^i base cells, $i = 1, 2, \dots$, are apparent, but may possibly disappear if different criteria are adopted when deciding whether to merge or not pre-existing halos and form a new structure, as well as if set of grids displaced in a different way (no longer by half a block-length) are used.

The distribution of formation redshifts is in good agreement with analytic predictions for high-mass halos, even if the peak of formation is systematically shifted towards more recent epochs. For intermediate mass objects, the MCM correctly reproduces the analytic expectations only at high redshifts, then it keeps forming halos also at very recent times, in contradiction with the EPS theory. Once more, a failure of the model in describing the history of low mass halos is evident, since they systematically form at earlier epochs than predicted.

Finally, a remarkable agreement of the two-point correlation function is found with respect to the predictions derived from the EPS theory (MW96; CLMP), for all considered masses and redshifts. This ensures that the model reliably retains information about the spatial correlation of high mass halos ($M \gtrsim M_*$), but it also overestimates the clustering of small objects (as do all analytic formulae). The amplitude of their correlation function in fact is significantly (2–3 times) higher than that predicted by the fitting formula recently proposed by Jing (1999), that correctly describes the correlation function found in numerical simulations. As discussed by Jing (1998, 1999) and PCL, this difference between N-body and EPS results in Lagrangian space, suggests that the criteria adopted in the PS theory for identifying bound virialised objects in the initial conditions are inadequate. The assumption of spherical symmetry for the collapse is certainly a strong simplification, and it also affects both the mass function and the typical formation epoch of structures. Considering that halos in the MCM are produced with a large variety of shapes, we intend to adopt a nonspherical condition and study its effects on the resulting mass function and formation redshift distribution in a future paper.

The physical processes ruling gas cooling, dissipative collapse, star formation, evolution and feedback (as well as interactions and merging between galaxies) are currently implemented in merging history trees of DM halos, so far obtained through two main approaches. In semi-analytic models (KW93; Kauffmann, White & Guiderdoni 1993; Kauff-

mann, Guiderdoni & White 1994; Cole et al. 1994; Baugh et al. 1998; Somerville & Primack 1999; and papers in these series), the merging history of DM halos is built through Monte-Carlo realisations of the block model or EPS formalism, with no or not accurate spatial information. More recently (Kauffmann, Nusser & Steinmetz 1997; Governato et al. 1998; Benson et al. 1999), DM halos have been selected from cosmological N-body simulations, but their merging trees are still computed with the Monte Carlo technique. In a “fully” hybrid model (Roukema et al. 1997; Kauffmann et al. 1999), merging trees are also computed from the output of large N-body simulations, and as a consequence they retain the spatial and dynamical information of the parent simulation, but they suffer from its limited mass resolution and expensive CPU cost.

The interest of the MCM is that it represents an intermediate approach. It is very fast, and it partly retains spatial information in the linear or weakly non-linear regime. A priori, it suffers from the same resolution problem as merging trees built from N-body simulations. For the same choice of cosmological parameters and box length, the 256^3 base cells have the same mass as the 256^3 particles, and reliable halos cannot be obtained below ~ 8 base cells or 10 particles. However, its low cost in terms of CPU time allows to run realisations of sub-boxes, thus improving the mass resolution. Moreover, many choices of the cosmological parameters, shape and normalisation of the power spectrum of linear fluctuations can be tested. So the MCM appears as a versatile and rapid method to test physical ideas about galaxy, group and cluster formation in various cosmologies, mostly when some degree of spatial information can be useful.

In particular, the MCM can be suitable for studying galaxy clusters, mainly at low redshifts, where a good agreement between MCM and analytic results is found, not only in terms of mass functions, but also in the distribution of formation redshifts, as well as in the halo two-point correlation function. Also the population of Lyman-break galaxies at $z = 3$ can be reasonably well studied by means of the MCM. In fact, these objects are often interpreted as star-forming galaxies located at the centre of halos of about $10^{12} M_\odot$ (e.g., Steidel et al. 1996; Giavalisco, Steidel & Macchetto 1996; Steidel et al. 1998; Giavalisco et al. 1998; Baugh et al. 1998; but see also Somerville, Primack & Faber 1998). For these masses and redshifts, the model provides a reasonably good description of both the mass distribution and the formation history. Moreover, the correlation function fairly matches the numerical results over a large range of halo separations, thus allowing in principle to investigate the clustering properties of Lyman-break galaxies.

5 CONCLUSIONS

The Merging Cell Model originally proposed by Rodrigues & Thomas (1996) for a scale-free power spectrum, has been developed in the case of the SCDM cosmology. Its reliability has been tested not only in terms of the halo mass function, but also comparing the distributions of the progenitor and child masses, as well as that of halo formation times, to the analytic predictions derived by the Press & Schechter theory and its extensions.

For the first time in the case of a semi-analytic merging tree model, we have also computed the halo two-point correlation function, and compared it to the available theoretical predictions.

We have stressed the major successes of the model, as well as its main weakness, and several possible solutions to improve it have been proposed.

Two main fields where the use of this method can be of particular interest have been recognised. It appears to be a suitable tool for studying the properties of cluster-scale objects, mainly at low redshift, as well as the population of Lyman-break galaxies, and their clustering at high z .

We intend to apply the method in a more realistic cosmological scenario (as the open and the lambda CDM), and directly test it against N-body simulations in a forthcoming work.

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